

An Electromagnetic Gauge Technique for Measuring Particle Velocity of Electrically Conductive Samples in Extreme Conditions

David X.P. Cheng^{*1}, Akio Yoshinaka²

Defence Research and Development Canada – Suffield

PO Box 4000, Station Main, Medicine Hat, Alberta T1A 8K6 Canada

^{*}david.cheng@drdc-rddc.gc.ca; ²akio.yoshinaka@drdc-rddc.gc.ca

Abstract

This paper presents an electromagnetic velocity (EMV) gauge technique which allows use of conductive samples in the measurement of material velocity subject to shockwave compression. A dynamic model of the induced current in the EMV gauge caused by the conductive samples moving in a uniform magnetic field is developed and the motional emf generated in the EMV gauge is extracted from the measured signal. Experimental results are presented and discussed to demonstrate the effectiveness of the technique.

Keywords

Electromagnetic Velocity Gauging; Shockwave; Particle Velocity

Introduction

Piezo-resistive film gauges are used regularly to determine normal stress in matter under shock compression. The functionality of these gauges relies on a pre-calibrated relationship between the change in relative resistance and the change in normal stress. Under extreme conditions, both substrate and adhesive constituents of these gauges can undergo phase and/or chemical changes whose effects on the overall performance of the gauge are not clear (Rosenberg, 2009). Electromagnetic velocity (EMV) gauges are another class of film gauges which permit the direct in-situ measurement of shocked material flow velocity. The active sensing element does not need pre-calibration; rather, this method requires exposure of the moving element to a well quantified external static magnetic field in order to produce motional electromotive force (emf). While applications of this technique for measuring velocities of shocked materials have been widely reported in the literature (Sheffield, 1999; Fritz, 1973), it is limited to non-metallic materials only. This is due to signal distortion caused by the induced currents within electrically conductive materials moving in an externally applied magnetic field at high velocities. To use an EMV gauge

with conductive samples, the motional emf generated in the pickup foil has to be extracted from the measured signal which results from the superposition of both motional emf and voltage from induced currents.

In this paper an electromagnetic technique is developed which uses an analytical model to estimate the dynamics of induced current between the copper substrate and the EMV gauge. The former is modelled as a non-magnetic conductor moving in a uniform magnetic field with a uni-directional translational velocity. The Foucault current in the moving conductor is discretized as a piece-wise constant signal. For a mm thick copper sample disk moving in a static magnetic field with a uniform translational velocity, the volume conductor is modelled as a magnetic dipole loop carrying a current and the EMV gauge is modelled as a closed circuit loop moving in the field of the magnetic dipole. The field produced by the magnetic dipole is calculated throughout the space in which the pickup foil gauge moves. Equations of mutual induction are derived and the induced current in the EMV gauge loop is solved numerically, which allows separation of the emf signal for measurement of the particle velocities. Numerical analysis is provided for the induced current in the EMV gauge with respect to the Foucault current in a copper sample undergoing a step wise change in velocity. A few experiments were performed in an attempt to demonstrate the feasibility of the proposed technique to compensate for signal distortion in measuring particle velocities of shock compressed copper. The ability to use EMV gauges within samples of high conductivity complements our use of manganin film stress gauges to quantify shocked states in matter.

EMV Gauge Modelling

Fig. 1 shows the setup for the EMV gauging technique

described in this paper. A shock wave is transmitted into a sample, comprising a copper disk above and below an insulated metallic foil element, where both copper and foil element are set in motion and, under the influence of an external magnetic field, generate a signal that is proportional to their velocity. According to Faraday's law, the particle velocity of the sample is given by $v = Bl/V_1$, where V_1 is the voltage across the active foil element, B is the average magnetic flux density as measured with a Gaussmeter at the location of the gauge, l is the length of the active element and v is the velocity of the gauge.

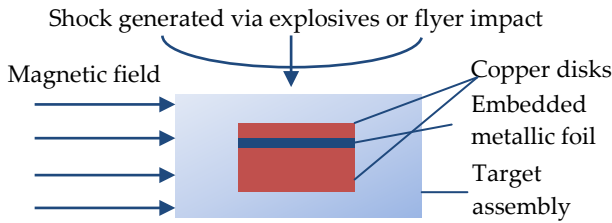


FIG. 1 EMBEDDED ELECTROMAGNETIC VELOCITY GAUGES WITH EXTERNALLY APPLIED MAGNETIC FIELD

A Model of the Induced Magnetic Field

In this paper, only the copper disk above the gauge is modeled as a loop of radius a carrying a current I_1 and moving along the z -axis with a constant translational velocity v toward the EMV gauge, as shown in Fig 2. The effect of induced current from the copper disk downstream from the gauge is for now neglected for simplicity.

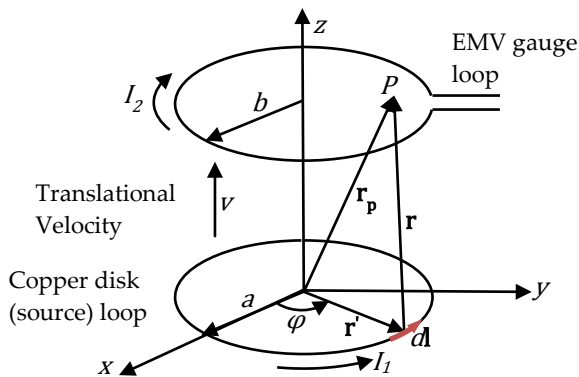


FIG. 2 MODELS FOR THE EMV GAUGE AND COPPER DISK

By the Biot-Savart law, the magnetic field at a point P on the surface of the EMV gauge loop is

$$\mathbf{B} = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\mathbf{l} \times \mathbf{r}}{r^3} \quad (1)$$

where

$$\mathbf{r} = -a \cos \varphi \mathbf{i} + (y - a \sin \varphi) \mathbf{j} + z \mathbf{k} \quad (2)$$

with a magnitude

$$r = (a^2 + y^2 + z^2 - 2ay \sin \varphi)^{1/2} \quad (3)$$

The differential current loop element satisfies

$$d\mathbf{l} = -a \sin \varphi d\varphi \mathbf{i} + a \cos \varphi d\varphi \mathbf{j} \quad (4)$$

The cross product $d\mathbf{l} \times \mathbf{r}$ becomes

$$d\mathbf{l} \times \mathbf{r} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a \sin \varphi d\varphi & a \cos \varphi d\varphi & 0 \\ -a \cos \varphi & y - a \sin \varphi & z \end{vmatrix} = ad\varphi [z \cos \varphi \mathbf{i} + z \sin \varphi \mathbf{j} + (a - y \sin \varphi) \mathbf{k}] \quad (5)$$

From eq. (1) the magnetic field at a point P on the EMV gauge loop surface is

$$B_x = 0 \quad (6)$$

$$B_y = \frac{\mu_0 I_1 a z}{4\pi} \int_0^{2\pi} \frac{\sin \varphi d\varphi}{(a^2 + y^2 + z^2 - 2ya \sin \varphi)^{3/2}} \quad (7)$$

$$B_z = \frac{\mu_0 I_1 a}{4\pi} \int_0^{2\pi} \frac{(a - y \sin \varphi) d\varphi}{(a^2 + y^2 + z^2 - 2ya \sin \varphi)^{3/2}} \quad (8)$$

B_y and B_z can be expressed as elliptic integrals and computed numerically. On the z -axis where $y=0$, B_y and B_z are

$$B_y = 0, \quad B_z = \frac{\mu_0 I_1 a^2}{2(a^2 + z^2)^{3/2}} \quad (9)$$

Models of the Copper Disk and EMV Gauge Loops

Let the mutual inductance of the copper disk and the EMV gauge loops be M_{12} . The circuit equations for the two loops can be expressed as

$$R_1 I_1 + L_1 \frac{dI_1}{dt} + \frac{dM_{12}}{dt} I_2 + M_{12} \frac{dI_2}{dt} = V_1 \quad (10)$$

$$L_2 \frac{dI_2}{dt} + R_2 I_2 + \frac{dM_{12}}{dt} I_1 + M_{12} \frac{dI_1}{dt} = 0 \quad (11)$$

In a configuration where the distance between the copper disk and the EMV gauge is much smaller than the radius of the current loops (i.e., $z \ll a$), the magnetic field generated by the source loop can be approximated by the magnetic density on the z -axis, given by eq. (9), and the magnetic flux going through the EMV gauge loop can be simplified as

$$M_{12} I_1 = B_z(0,0,z)(\pi b^2) = \frac{\mu_0 \pi I_1 a^2 b^2}{2(a^2 + z^2)^{3/2}} \quad (12)$$

And the mutual inductance can be expressed as

$$M_{12} = \frac{\mu_0 \pi a^2 b^2}{2(a^2 + z^2)^{3/2}} \quad (13)$$

Thus the total time derivative of M_{12} is

$$\frac{dM_{12}}{dt} = \frac{dM_{12}}{dz} \cdot v = -\frac{3\mu_0\pi a^2 b^2 v}{2} \cdot \frac{z}{(z^2 + a^2)^{2.5}} \quad (14)$$

where v is the moving velocity of the copper disk toward the EMV gauge. To simplify the problem, we assume that the current running around the copper disk loop I_1 holds a constant value in a short time interval so that eq. (11) becomes

$$L_2 \frac{dI_2}{dt} + R_2 I_2 = -\frac{dM_{12}}{dt} I_1 \quad (15)$$

Applying eq. (14), we get

$$L_2 \frac{dI_2}{dt} + R_2 I_2 = I_1 \cdot \frac{3\mu_0\pi a^2 b^2 v}{2} \cdot \frac{z}{(z^2 + a^2)^{2.5}} \quad (16)$$

To perform a modal analysis for the induced current I_2 , we define the following dimensionless ratios

$$\tau = \frac{v}{a} t, \rho = \frac{L_2 v}{a R_2}, I = \lambda \frac{I_2}{I_1}, \lambda = \frac{2a L_2}{3\mu_0 \pi b^2} \quad (17)$$

Substituting (17) into (16), we derive the modal equation for the induced current in terms of the ratio I

$$\frac{dI}{d\tau} + \frac{1}{\rho} I = \frac{\tau}{(\tau^2 + 1)^{2.5}} \quad (18)$$

which can be solved numerically as shown in Fig. 3. From the numerical solutions of the induced current, we have observed that

- The Foucault currents in the copper disk moving with a uniform velocity induce currents on the EMV gauge before the gauge starts to move.
- The induced current causes a reversal of the emf signal with a positive overshoot.
- The positive overshoot extends for a longer time with higher copper disk velocity.

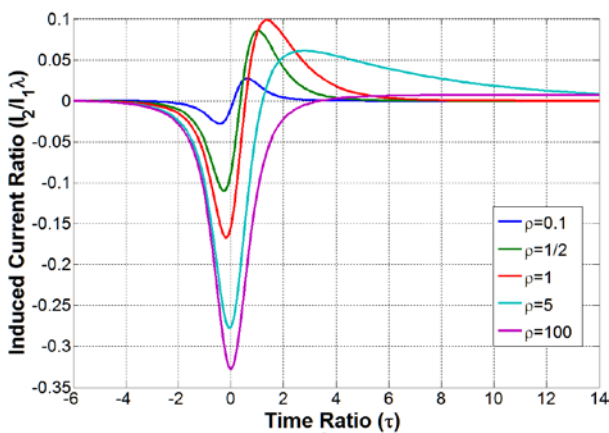


FIG.3 MODES OF INDUCED CURRENTS IN THE EMV GAUGE

Modal Analysis

The procedures for applying modal analysis to the

experimental data for signal extraction are as follows:

- Step1: Use a Fast Fourier Transform to low-pass filter the raw experimental data in order to get a smoothed curve so that the mode shapes are evident.
- Step2: Determine the boundary values of the emf that corresponds to the modes of the induced current demonstrated in Fig. 3.
- Step 3: Solve eq. (18) as a boundary value problem and obtain a modal solution.
- Step 4: Apply the modal to the smoothed experimental data by means of superposition.
- Step 5: Repeat Steps 2 to 4 until the mode shape is not evident on the curve of the experimental data.
- Step 6: Combines all the modals solved from Steps 2 to 5 to construct a complete induced emf profile and extract the motional emf from the raw signal.

Experimental Tests, Results and Discussion

In order to test the performance of the models of signal extraction for EMV gauging in shock wave environments, a number of trials were performed whereby composition C-4 military explosive, hand packed in a PVC tube (52mm i.d. x 4.2mm wall x 203mm high for test 275A and 20.2mm i.d. x 3.3mm wall x 101mm high for test 295A) was used to generate an incident shock wave in copper.

The EMV gauges used for the tests were manufactured in-house (Fig. 4 (a)). A 25μm thick brass foil was cut with an active length of 10 mm, which produced motional emf when moving through a magnetic field. The foils were encapsulated with 250μm (test 295A) or 500μm (test 275A) thick Teflon adhesive sheets on both sides. The completed gauges were then embedded between two copper disks (31.75 mm dia. x 3.09 mm thick above and 22 mm thick below for test 295A, and 57.05 mm dia. x 3.09 mm thick above and 10 mm thick below for test 275A) using liquid epoxy adhesive, as shown in Figs. 4(b) and (c).

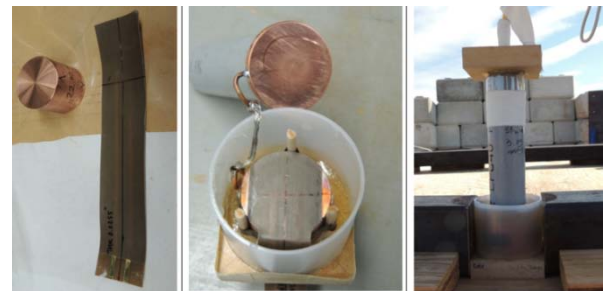


FIG. 4 (A) IN-HOUSE MADE EMV GAUGE AND (B) EMV GAUGE IN THE TARGET SAMPLE (C) DONOR/TARGET ASSEMBLY

For trial 275A, a $75\ \Omega$ resistor was connected in series with the EMV gauge and terminated with $75\ \Omega$ on a single scope channel for impedance matching of the whole gauge circuit loop. For trial 295A, each lead of the EMV gauge was connected to a different oscilloscope channel via the center conductor of a 1.5 m long RG-179 coaxial cable, resulting in a differential input to reject common mode noise. Each channel was terminated with $75\ \Omega$ and the voltage difference between these channels provided us with a measure of voltage across the foil, accounting for resistive losses across cables. The top and bottom copper disks were grounded to a central grounding point at the instrumentation bunker for all tests.

An electromagnet built in-house was used to provide an external static magnetic field. The magnet essentially consisted of 50.8 mm square A36 mild steel bars used as pole pieces, two of which were wrapped with #8 AWG cable. A switching DC power supply was used to provide the coils with a constant current.

Calibration of the electromagnet was performed before every trial. The variation of magnetic flux density (B) with vertical position along the center of the pole piece gap where EMV gauges were located was measured, and an average flux density was used for simplicity. The variation in B along the sensing width of the EMV foil was neglected, since lateral fluctuations in the magnetic field were within better than 1% over the gauge width.

The EMV gauge model developed in this paper is applied to the velocity data of the copper samples acquired in two of the EMV gauging trials. A separate experiment was performed to determine the normal stress at the location of the EMV gauge within the target sample for trial 295A using a Dynasen Inc. MN10-0.050-EPTFE manganin gauge. The copper material velocity was then calculated via the known Hugoniot data (Marsh, 1980) given the normal stress at the EMV gauge location. The measured peak normal stress was 20.6 GPa which corresponds to a particle velocity of 0.50 mm/ μ sec.

From the raw experimental data (Figs. 5 and 8), we can observe that in both trials the waveforms first show a negative peak with a magnitude ranging between 0.34–0.41 mm/ μ sec. The EMV gauge model developed in this paper suggests that the moving top copper disk perturbed the magnetic field and induced a current in the EMV gauge circuit before the gauge produced motional emf. This is followed by a positive peak,

where the motional emf from the EMV gauge is embedded.

The modal analysis is performed on both waveforms using the procedures described in last section. Twenty-six modals (Fig. 6) for the data of trial 295A, and thirty-five modals (Fig. 9) for trial 275A, were solved at various time instants with suitable strength (i.e., satisfying the boundary values), and then were applied to the waveforms to compensate for the negative signal distortion and positive signal perturbation on the EMV gauge. It should be noted that all the EMV gauge model outputs are shown in terms of velocities that are converted from the corresponding induced current in the EMV gauge. The results are shown in Figs. 7 and 10. The negative reversal signals were nearly eliminated and the positive signals were also compensated for the extended effect of the induced currents. In Fig. 7, the extracted EMV gauge signal shows an average peak value of 0.53 mm/ μ sec for trial 295A, which matches the peak value of 0.50 mm/ μ sec inferred by the manganin gauge for trial 295A. The charge diameter for trial 275A was about 2.6 times larger than and twice as long as the donor charges in all other tests. A higher average peak value of 0.72 mm/ μ sec particle velocity is observed from the output of the EMV gauge model for trial 275A, as shown in Fig. 10, which appears to confirm the expected greater delay in both lateral and rear expansion waves. These values suggest that the EMV gauge model provides a correct measure of the incident particle velocity.

It can be also observed in Figs. 7 and 10 that the EMV gauge model increases the period over which an effective measurement of particle velocity can be made. In trial 295A the effective measurement time was extended from about 0.15 to 0.26 μ sec, whereas in trial 275A it was extended from about 0.2 to 1 μ sec.

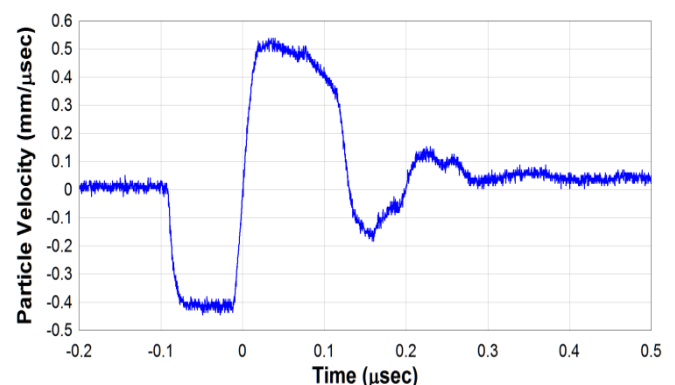


FIG. 5 PARTICLE VELOCITY RAW DATA CONVERTED FROM EMF MEASURED BY THE EMV GAUGE IN TRIAL 295A

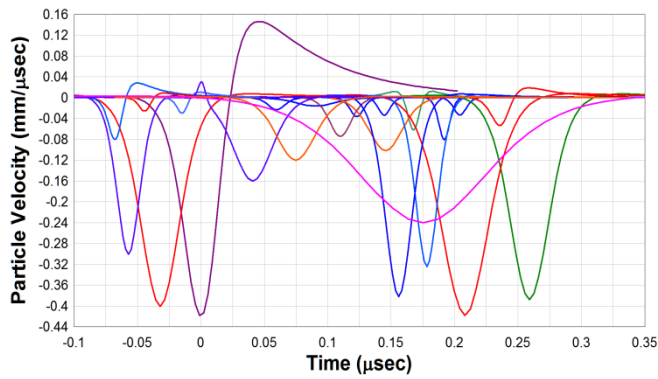


FIG. 6 CONVERTED MODES OF INDUCED EMF SOLVED FROM THE INDUCED CURRENT MODEL FOR TRIAL 295A

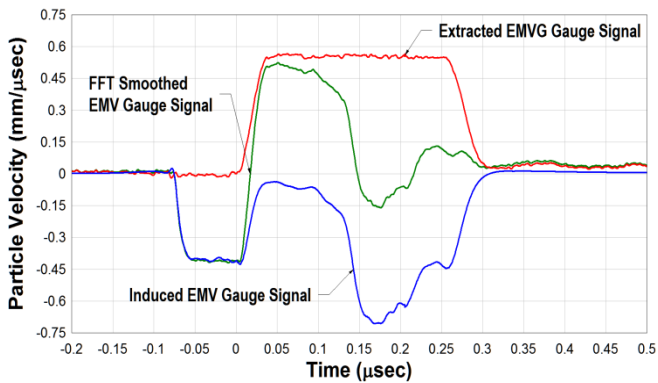


FIG. 7 COMPARISON OF THE INPUT, INDUCED AND EXTRACTED EMV GAUGE SIGNALS FOR TRIAL 295A

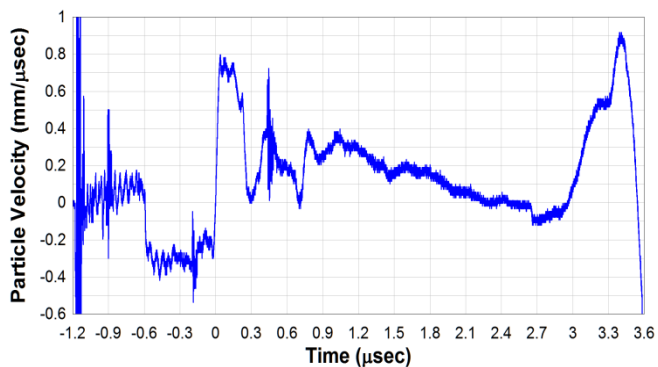


FIG. 8 RAW DATA OF THE PARTICLE VELOCITY CONVERTED FROM EMF MEASURED BY THE EMV GAUGE IN TRIAL 275A

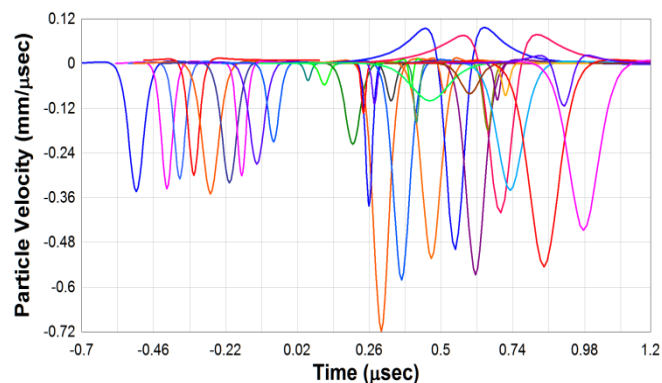


FIG. 9 CONVERTED MODES OF INDUCED EMF SOLVED FROM THE INDUCED CURRENT MODEL FOR TRIAL 275A

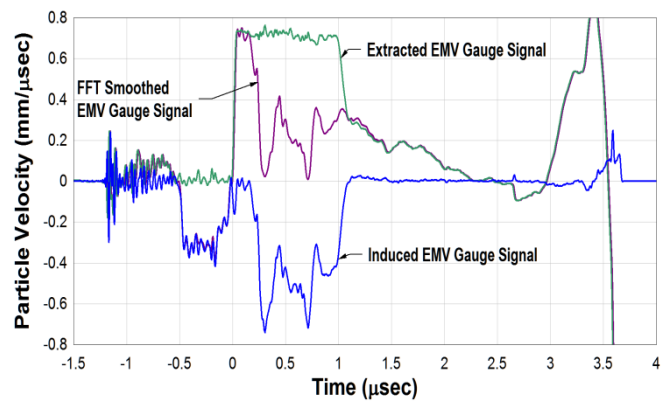


FIG. 10 COMPARISON OF THE INPUT, INDUCED AND EXTRACTED EMV GAUGE SIGNALS FOR TRIAL 275A

Conclusions

A model was developed in attempt to discriminate motional emf from the resultant signal produced by an electromagnetic velocity (EMV) gauge embedded within shock compressed copper. While the current model only accounts for the effects of bulk motion of a copper sample on the upstream (relative to the direction of shock propagation) side of the EMV gauge, the results suggest that it is possible to eliminate the negative signal distortion induced by a moving conductor in an external magnetic field, thereby extending the measurement of particle velocity history (up to 1 μsec for the cases examined in this paper). Although the EMV gauge technique developed in this paper requires further development to account for the fact that a moving conductor was also located downstream of the gauge, we expect that a similar model designed to capture a bulk copper volume moving away from the foil gauge could further extend the useful part of the velocity signal. Further model refinement will be needed to confirm this.

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